F-TEST

The F-test is named in honour of the great statistician R.A. Fisher. The object of the F-test is to find out whether the two independent estimates of population variance differ significantly, or whether the two samples may be regarded as drawn from the normal populations having the same variance, for carrying out the test of significance, the F-ratio can be calculated as;

1.

$$F=\frac{S_1^2}{S_2^2}$$

Where, $$S_1^2=\frac{\Sigma(X_1-\overline{X}_1)^2}{n_1-1}$$

$$S_2^2=\frac{\Sigma(X_2-\overline{X}_2)^2}{n_2-1}$$

It should be noted that S_1^2 is always the larger estimate of variance, *i.e.*, $S_1^2 > S_2^2$

2.

$$F=\frac{v_1}{v_2}$$

Where $v_1 > v_2$

v_1 = variance of first observation

v_2 = variance of second observation

3. F ratio can also be calculated by following formula

$$F=\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$$

Where, S_1 – Standard deviation of sample drawn from population 1

S_2 – Standard deviation of sample drawn from population 2

σ_1- Standard deviation of population 1

σ_2- Standard deviation of population 2

The calculated value of F is compared with the table value for V_1 and V_2 at 5% or 1% level of significance. If calculated value of F is greater than the table value than the F-ratio is considered significant and the null hypothesis is rejected. On the other hand, if the calculated value of F is less than the table value the null hypothesis is accepted and it is inferred that both the samples have come from the population having same variance.

Since , F test is based on the ratio of two variances, it is also known as the Variance Ratio Test. The ratio of two variances follows a distribution called the F- distribution named after the famous statistician R.A. Fisher.

Assumptions in F-test

1. Normality, *i.e.*, the values in each group is normally distributed.
2. Homogeneity, *i.e.*, the variance within each group should be equal for all groups.
3. Independence of error, *i.e.*, the error (variation of each value around its own group mean) should be independent for each value.

Application of F-test

ANALYSIS OF VARIANCE (ANOVA)

It is statistical technique specially designed to test whether the means of more than two quantitative populations are equal.

Box 7.14 Problem: Suppose for instance, we are estimating the amount of phosphorus in water samples by two different methods, one is a standard method and the other is a modified one. From the two replicate samples determine whether the variance of the modified method differs significantly from the standard method.

Phosphate mg l^{-1}

Standard method X_1	X_1^2	modified method X_2	X_2^2
0.27	0.0729	0.30	0.9000
0.25	0.0625	0.28	0.0784
0.23	0.0529	0.31	0.0961
0.30	0.0900	0.29	0.0841
0.31	0.0961	0.27	0.0729
0.26	0.0676	0.25	0.0625
0.29	0.0841	0.28	0.0784
$\Sigma X_1 = 1.91$	$\Sigma X_1^2 = 0.5261$	$\Sigma X_2 = 1.98$	$\Sigma X_2^2 = 0.5534$

$$SS_1 = \Sigma X_1^2 - \frac{(\Sigma X_1)^2}{n} = 0.5261 - \frac{(1.91)^2}{7} = 0.5261 - 0.5211 = 0.005$$

$$s_1^2 = \frac{0.005}{7-1} = .0083$$

$$SS_2 = 0.5534 - \frac{(1.98)^2}{7} = 0.5624 - 0.5600 = 0.0024 \quad s_2^2 = \frac{0.0024}{6}$$

$$= 0.0004 \qquad F = \frac{s_1^2}{s_2^2} = \frac{0.00083}{0.0004} = 2.075$$

The tabulated F value for $v_1 = 6$ and $v_2 = 6$ is 4.28. Since the calculated value is less than this, we conclude that there is no significant difference in the precision of the two methods.

Table 7.2 Value of F at the 95 percent confidence level

ν_1 =	1	2	3	4	5	6	7	8	9	10	15	20
ν_2												
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.40	2.33
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12